Fundamental Journals
International Journal of Fundamental Physical Sciences (IJFPS) ISSN:2231-8186
IJFPS, Vol 4, No 4, pp 111-113, Dec , 2014 **R. Alemi**
DOI:10.14331/ijfps.2014.330074
http://dx.doi.org/10.14331/ijfps.2014.330074

Determination of the Correlation Factor Between the Brownian Particle and Its Environment

Reza Alemi

Independent researcher, Damghan, Iran

E-mail: alemi_r@yahoo.com

(*Received Jun 2014; Published Dec 2014*)

ABSTRACT

The purpose of this brief extension is reporting the exact results, by introduction a correlation factor, for the dissipative system which consisting of a particle in one-dimension coupled to a sum of an infinite number of harmonic oscillators.

Keywords: correlation factor, dissipative, discontinuity, time delay

DOI:10.14331/ijfps.2014.330074

INTRODUCTION

In the nineteenth century, people envisaged how include dissipation in the quantum mechanical description of systems. The final aim was to establish the quantum theory of a system which is described in the classical limit by the Langevin equation (Ulrich Weiss, 2012) ,

$$M\ddot{q} + \eta\dot{q} + V'(q) = f(t) \quad (1)$$

Where q is the particle coordinate, $V(q)$ is an external potential and $f(t)$is a fluctuating force such that $\langle f(t)\rangle = 0$ and$\langle f(t)f(t')\rangle = 2\eta k_B T\delta(t-t')$. The quantity M is the mass of the particle, η denotes the damping constant or the friction coefficient and k_B the Boltzmann constant. Defining $V'(q) \equiv dV/dq$ and using dots for time derivatives. The origin of this problem lay in the fact that the standard procedures of quantization are based on the existence of either a Hamiltonian or a Lagrangian function for system that their equations are possible to obtain the Langevin equation.

It is well know that it is not possible to obtain a Langevin equation from the application of the classical Lagrange's or Hamilton's equations to any Lagrangian or Hamiltonian which has no explicit time dependence. The employment the first Hamiltonian function by (Caldirola, 1941) of time dependent functions would allow one to use the standard procedures of quantization directly but it would create problems with the uncertainty principle. Yet scientist have tried to solve this problem. These attempts fall into two main categories: They either look for new schemes of quantization or use the system-plus-reservoir (or bath) model. The simplest way to model the bath of oscillators was proposed by (Feynman & Vernon Jr, 1963) and also the simple model to study in detail the way dissipation arises from a quantum point of view was proposed by (A. Caldeira & Leggett, 1981).

We can study the dynamics of a free particle interacting with an environment or reservoir made up of an infinite number of independent harmonic oscillators in thermal equilibrium. In the case of a linear coupling, the effect of the environment can be eliminated and the particle's equation of motion can be established and it can take form of a generalized Langevin equation (Kubo, 1966; Ulrich Weiss, 2012). This model of dissipation is approach as mentioned in A. O. Caldeira and Leggett (1983). The Caldeira-Leggett Hamiltonian reads,

$$H = \frac{P^2}{2M} + V(q) + \sum_i \left(\frac{{p_i}^2}{2m_i} + \frac{1}{2}m_i{\omega_i}^2{q_i}^2\right) + q\sum_i C_i\, q_i + q^2\sum_i \frac{{C_i}^2}{2m_i{\omega_i}^2} \quad (2)$$

Copyright © 2014 *Fund Jour.*

The first two terms correspond to the Hamiltonian of a quantum particle of mass M and momentum P in an external potential V at position q. The third term describes the bath of infinite independent harmonic oscillators of masses m_i, described by the characteristics q_i and the conjugate momenta p_i. The correlation factors C_i are coefficients which depend on the details of the coupling. The last term is a counter- term which must be included to ensure that dissipation is homogeneous in all space. If fluctuating force is used, term $-qf(t)$ will add to the Hamiltonian. The equations of motion of a global system described by the Hamiltonian above read.

$$M\ddot{q} + V'(q) + q\sum_i \frac{C_i^2}{m_i\omega_i^2} = \sum_i C_i\, q_i \tag{3}$$

$$m_i\ddot{q}_i + m_i\omega_i^2 q_i = C_i q \tag{4}$$

The equation of motion of the particle coupled with the bath, according to Eqs (3) and (4), is in the form of a closed,

$$M\ddot{q}(t) + V'(q(t)) + \frac{2}{\pi}\frac{d}{dt}\int_0^t \int_0^\infty q(t')\frac{J(\omega)}{\omega}\cos[\omega(t - t')]\, d\omega dt' = -q(t)\sum_i \frac{C_i^2}{m_i\omega_i^2} + F(t) \tag{5}$$

Defining $V'(q(t)) \equiv dV/dq$ and the function $F(t)$ involved in Eq (5) is defined in terms of the microscopic parameters of the model by the formula,

$$F(t) = \sum_i C_i[q_i(0)\cos\omega_i(t) + \frac{p_i(0)}{m_i\omega_i}\sin\omega_i(t)] \tag{6}$$

The variables $q_i(0)$ and $p_i(0)$ associated with the initial state of the oscillators of bath. To provide a good description of the dissipation mechanism, a relevant quantity is the bath spectral density of the coupling, introduced as follows,

$$J(\omega) = \frac{\pi}{2}\sum_i \frac{C_i^2}{m_i\omega_i}\delta(\omega - \omega_i) \tag{7}$$

Note, in the infinite number of harmonic oscillators of the bath, the spectral density function can be seen as a continuous function. The spectral density function provides a constraint in the choice of the coefficients C_i. The relation between ω and $J(\omega)$ can be a generic form $J(\omega) \propto \omega^s$. Where s is constant and its different values to a different behavior. If $0 < s < 1$ is sub-ohmic, if $s > 1$ is super-ohmic and in finally when this function has the form $J(\omega) = \eta\omega$, the corresponding classical kind of dissipation is ohmic (Da Costa, Caldeira, Dutra, & Westfahl Jr, 2000), in this case the Langevin Eq (1) is approximately found.

The aim of this work is reporting the exact results, by introduction a new correlation factor and or a new spectral density function.

NEW SPECTRAL DENSITY FUNCTION

In this work, the spectral density function of the coupling with the bath is introduced the form,

$$J(\omega) = \eta\omega\cos(\omega\varepsilon) \tag{8}$$

That has not been properly defined for in previous works. While ε is not determined at this stage, it can be told some of the requirements that must satisfy. Firstly, in fact involve the time dimension; secondly, vanish if $\varepsilon = 0$, since then there is no dissipative process; thirdly, $\varepsilon > 0$, at this stage, using the new spectral density function in the third term of the integro-differential Eq (5) gives,

$$\frac{2}{\pi}\frac{d}{dt}\int_0^t \int_0^\infty q(t')\frac{J(\omega)}{\omega}\cos[\omega(t - t')]\, d\omega dt = \frac{d}{dt}\int_0^t q(t')\left(\delta(t + \varepsilon - t') + \delta(t - \varepsilon - t')\right)dt \tag{9}$$

Now, the necessity of new constraint to the application of the coefficients C_i. To proceed technical step mathematically, the coupling coefficients or the correlation factors obtain,

$$C_i^2 = \frac{2}{\pi}\eta m_i\omega_i^2\cos(\omega_i\varepsilon)\frac{d\omega_i}{di} \tag{10}$$

The last term from the Hamiltonian (2) is a counter-term which must be included to ensure that dissipation is homogeneous in all space. As the bath couples to the position, if this term is not included the Hamiltonian is not translationally invariant, in the sense that the coupling is different wherever the quantum particle is located. When the coefficients C_i have form relation (10), the counter term by force is zero. One then proceeds to substitute Eq (9) and (10) into (5), we can write the equation of motion for the particle of the coupling with the bath which takes the similar form of the Langevin equation,

$$M\ddot{q}(t) + V'(q(t)) = F(t) \qquad (\text{for } t \le \varepsilon) \tag{11}$$

$$M\ddot{q}(t) + V'(q(t)) + \eta\dot{q}(t - \varepsilon) = F(t) \qquad (\text{for } t > \varepsilon) \tag{12}$$

RESULT AND DISCUSSION

Now we can introduce ε the as time delay in the frictional term. Likewise, there is a discontinuity in the acceleration of the particle at $t = \varepsilon$, and also a discontinuity in the equations of motion. If the delay time appeared in the frictional term, not considered; the dissipative in Eq (12), namely $\eta\dot{q}(t - \varepsilon)$, will be removed.

CONCLUSION

In this paper have been applied a new bath spectral density function. The value $J(\omega) = \eta\omega\cos(\omega\varepsilon)$ is especially important. Indeed, in this case, the equation of motion of the particle coupled with the bath contains a friction term proportional to the velocity together the delay. In the first time, the coefficients which depend on the details of the coupling have been obtained and in finally, the discontinuity in equations of motion of a dissipative particle has been shown.

REFERENCES

Caldeira, A., & Leggett, A. (1981). Influence of dissipation on quantum tunneling in macroscopic systems. *Physical review letters, 46*(4), 211.

Caldeira, A. O., & Leggett, A. J. (1983). Path integral approach to quantum Brownian motion. *Physica A: Statistical mechanics and its Applications,121*(3), 587-616.

Caldirola, P. (1941). Forze non conservative nella meccanica quantistica. *Il Nuovo Cimento, 18*(9), 393-400.

Da Costa, M. R., Caldeira, A., Dutra, S., & Westfahl Jr, H. (2000). Exact diagonalization of two quantum models for the damped harmonic oscillator. *Physical Review A, 61*(2), 022107.

Feynman, R. P., & Vernon Jr, F. (1963). The theory of a general quantum system interacting with a linear dissipative system. *Annals of physics, 24*, 118-173.

Kubo, R. (1966). The fluctuation-dissipation theorem. *Reports on Progress in Physics, 29*(1), 255.

Ulrich Weiss. (2012). *Quantum Dissipative Systems* (t. Edition Ed.).

Copyright © 2014 *Fund Jour.*