A generalized metric for the average distribution of matter on the cosmological scale: toward a clear and flat universe?

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Abstract

Following on from our previous works on cosmology, we propose a generalized metric, denoted G, applicable to the universe as a whole, treated as homogeneous and filled with matter at its average density. Using the weak-field approximation, the individual Schwarzschild terms M/r are extended to a continuous cosmological medium, yielding a global gravitational potential proportional to $\rho_u R_u^2$, where ρ_u is the mean cosmic density and R_u the Hubble radius. The resulting metric can be interpreted through an effective cosmological refractive index of vacuum. For a critical homogeneous universe, this index is exactly 2. Taking into account large-scale inhomogeneities and the nonlinear dependence of the index on density increases its value, possibly up to about 2.4. As suggested in our previous works, a cosmological light speed reduced by a factor of about 2.4 relative to its local value c_0 offers a unified explanation for several phenomena commonly attributed to dark matter and dark energy, etc.. Incorporating the gravitational potential generated by the total mass of the universe into the metric also provides a new perspective on Mach's principle: inertia appears as a response to the influence of the universe as a whole. The equivalence of inertial and gravitational mass follows naturally in a critical universe, where the global potential is of order c_0^2. In this framework, cosmic flatness emerges from an extended Machian relation rather than from inflation, opening the way to alternative cosmological models without dark matter or dark energy.

Keywords: Cosmology; Mach's principle; critical density; gravitational potential; cosmological refractive index; inertia; speed of light; G-metric

Introduction

In two recent articles (Guy, 2024, 2025a), we examined the matter and energy content of the universe (95% of which appears to us to be unknown), as well as the various “tensions” encountered by astrophysicists. The latter tackle these difficulties in ways that are independent of one another. On our part, however, we have proposed a unifying research approach: to consider that, on a cosmological scale, light might have a “speed” lower than its value in the “local” vacuum (our laboratories, our solar system). It could be divided by a factor of approximately 2.4: this allows us to resolve, on average and through a single solution, the dozen or so problems or tensions that constitute the dark side of contemporary cosmology. We have proposed the calculation of an equivalent refractive index on the cosmological scale, called n_c[1]. The calculation first involves a metric understood in general relativity (the Schwarzschild metric, hereafter denoted by the letter S), then generalizes the contribution of a single mass term to a continuous cosmological medium. After this was written, it became apparent to us that the initial assumptions (related to the formulation of the S metric) needed to be adapted for the calculation of n_c on the scale of the universe[2].

In this article, we aim to present this adaptation. We explain the assumptions that allow us to incorporate individual contributions of the form M/r (taken into account in the metric S; the M’s are masses located at distances r from the observer), and to transform a local gravitational potential into an average cosmological potential. In doing so, *we derive an original metric*, in the generalized sense, which we call G[3]. Extending this approach, we examine the implications of this metric: the cosmological index is *identically equal to 2 for a universe near criticality*, such as ours, i.e., a speed of light halved relative to its local value. The increase from 2 to a value of 2.4 is explained by the effect of the universe’s inhomogeneities (voids/structures) in the nonlinear relationship between the index and density. Furthermore, the inclusion in the metric of the gravitational potential due to the masses of the universe provides an operational perspective on Mach’s principle, Newton’s law of inertia, and the law of the equality of inertial mass and gravitational mass. The flatness of our universe is also relevant: this should not be viewed as a fortuitous coincidence, but as the expression of a structural property of the physical reality.

[1] The use of an index is not merely a "optical analogy" for general relativity; it is a genuine manifestation of how the theory works. We will denote by c or c_0 the value of the standard speed of light in a vacuum, and $c_c = c/n_c$ or c_0/n_c the value calculated on the cosmological scale within the framework of general relativity.
[2] In particular, with regard to the addition of potentials, in the *a priori* nonlinear context of general relativity.
[3] With G standing for “Gravitation,” “G-constant,” or even (!?) “Guy.”

At the outset of this article, it is worth making two remarks.

The first concerns the nuances to be considered when discussing the speed of light. What measurement should we rely on to determine its actual value between a distant star and us? Strictly speaking, such a speed has no physical meaning: it would require independently measuring both a distance and a travel time, which is impossible in this case[4]. Without delving into the details of our work, let us say that what we will call the speed of light refers, at a minimum, to the parameter c in the denominator of the ratio v/c: that of the Doppler effect formula and redshifts, carried by light from distant stars, and enabling various astronomical calculations. Or to the value of the parameter c involved in the equations of gravitational deflection effects. Etc. The fact remains that for us, the quantity c refers first and foremost to a propagation (to be understood relationally, comparing it to other propagations in terms of speed ratios) before being a "simple" structural constant.

The second point concerns the flatness property of our universe and the value of the parameter Ω, which relates the universe's matter-energy density to the critical density. This parameter depends on the chosen expansion model, defined in the standard model by its various components: baryonic matter, dark matter, and dark energy. Since we believe that these last two components are unnecessary, the entire Standard Model would need to be rewritten. What would then become of criticality? According to the authors, this property is demonstrated by robust tests and would be little affected by a change in model[5]. For different reasons that we will explain, we agree. Thus, as a first approximation, we will attempt to preserve flatness (now relative to baryonic matter alone) without a complete overhaul of the standard model of expansion. We must therefore work with the mathematical tools we have chosen to use, which exhibit certain divergent aspects in the vicinity of critical universe values. This is a source of uncertainty that we are treating as such for the time being. The proposals in this article thus provide a provisional explanatory framework and *orders of magnitude* for the physical and cosmological parameters under discussion, serving as a basis for future quantitative refinements. A practical consequence of this preliminary approach and its approximations will be, for example, the use of the "=" sign where full rigor would require the use of "≈" (roughly equal to) or "~" (of the same order of magnitude).

[4] And neither space nor time exists independently of one another or of the phenomena of the world (see our work, *e.g.*, Guy, 2022).
[5] In the standard model, $\Omega = 1$ to within 0.2%.

Our plan is as follows. In the first section, we will review the main points of our initial approach, explaining its initial assumptions (use of the metric S). In the second section, we will introduce the new metric G, along with the conventions underlying it. Next (Section 3), we will show that, for a universe at critical density, the cosmological index is equal to 2. In the fourth part, we will revisit Mach's principle, then (Part 5) discuss the flatness property of the universe. In the sixth part, we will see how the index can be increased to a value of 2.4 (while remaining in the critical configuration). In Part 7, we will examine how a model containing only baryonic matter can be compatible with existing density measurements in a critical universe. Finally, in Part 8, we will provide some points of comparison between the G metric and the Friedman-Lemaître-Robertson-Walker metric (hereafter abbreviated as FL or FLRW). We will conclude with a few remarks. In Appendix 1, we summarize the various astrophysical problems addressed in our previous works, for which the present approach appears relevant.

1. Deriving a cosmological refractive index: an introduction to our approach

The Karl Schwarzschild metric (1916) is suited to the effect of a point mass M located at a distance r from the observer. Its form is derived from Albert Einstein's equations with several assumptions, which we summarize as follows.

i) A spherical symmetry assumption: the matter density distribution ρ(r) is independent of θ and φ (r, θ, and φ are the coordinates in spherical coordinates). The metric is given by:

$$ds^2 = a(r)c_0^2dt^2 - b(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \qquad (1)$$

where t is time, c_0 is the speed of light in a vacuum, which we will call the local speed[6], and a and b are the coefficients of the metric.

[6] c_0 is part of the toolkit at our disposal: it allows us to estimate the projected distances from our Euclidean laboratories to celestial objects. As we have said, since it appears in numerous equations, it is often taken as a "structural" constant. Here we will consider it primarily as the expression of a velocity evaluated through a set of comparisons with other velocities (particularly those of gravitational origin, based on spatial distributions of mass) and allowing us to evaluate still others, such as that of light c_c on the cosmological scale (see Guy, 2025b, 2026). We have discussed the contexts in which we use c_0 (physics constructed first and locally) and those in which we use c_c (cosmological trajectories).

ii) A condition regarding the solution in the vacuum surrounding the mass. We solve Einstein's equations with $T_{\mu\nu} = 0$ (energy-momentum tensor) everywhere except at the center, where the mass M is located.

iii) A stationarity assumption. The gravitational field does not depend on time, i.e. $\partial_t g_{\mu\nu} = 0$, where the $g_{\mu\nu}$ are the coefficients of the metric (here, a and b).

iv) The assumption of a single central field: all gravity originates from the point mass M.

v) The use of appropriate coordinates: we choose coordinates where the angular geometry is Euclidean: $g_{\theta\theta} = r^2$.

These assumptions lead to the values of the coefficients a and b (see Schwarzschild, *op. cit.*)

$$g_{tt}(r) = a(r) = 1 - \frac{2GM}{rc_0^2} \tag{2}$$

$$g_{rr}(r) = b(r) = \left(1 - \frac{2GM}{rc_0^2}\right)^{-1} \tag{3}$$

Where G is the universal gravitational constant. The term GM/r is the opposite of the gravitational potential Φ, according to

$$\Phi = -\frac{GM}{r} \tag{4}$$

written in terms of a density ρ assumed to be uniform. The preceding relations read

$$g_{tt} = a(r) = 1 + \frac{2\Phi}{c_0^2} \tag{5}$$

$$g_{rr} = b(r) = (1 + \frac{2\Phi}{c_0^2})^{-1} \tag{6}$$

In our work, we have restricted the metric to

$$ds^2 = a(r)c_0^2 dt^2 - b(r)dr^2 \tag{7}$$

by eliminating the angular terms. This implicitly corresponds to a radial approximation. The condition $ds^2 = 0$ applies to the propagation of light (zero geodesics); it allows us to determine a speed c_S of light (S denoting Schwarzschild) in the space surrounding the mass M

$$c_S^2 = \frac{dr^2}{dt^2} = \frac{a}{b} c_0^2 \tag{8}$$

We can then define a gravitational refractive index n_S for radial light propagation[7]

$$n_S^2 = \frac{c_0^2}{c_S^2} = \frac{b}{a} \tag{9}$$

And

$$n_S = \left|\sqrt{\frac{b}{a}}\right| \tag{10}$$

The index is positive, hence the absolute value in relation (10)[8]. This interpretation is standard in relativity. The coefficients of the metric, as well as the index, are functions of the distance r from the center of the mass M, which is a given in the problem. One can also say that they are functions of the gravitational potential $\Phi = GM/r$, and would not change for a variation in the values of M and/or r that would respect the constancy of the ratio M/r[9].

2. A New Metric

In our work, the next step was to extend the previous results by replacing a point mass with a continuous cosmological distribution according to the scheme $\frac{GM}{r} \to \int \frac{G\,dM}{r}$ by summing gravitational potentials. We did this by carrying out calculations that led us away from the initial assumptions (the linearity of the summation appears, in particular, to contradict the nonlinearity of the equations of general relativity). We must therefore start over and emphasize that we are now deriving a new metric. We will call it G, constructed based on the metric S; we will write it as

$$ds^2 = a'(\beta) c_0^2 dt^2 - b'(\beta) dr^2 \tag{11}$$

With new coefficients a' and b' to be determined, where β is a parameter that will describe the possible characteristics of the universes, as they result from the integrations performed. For the summation of masses beyond a single point mass M to be valid, several assumptions (in addition to those made regarding the metric S) are necessary.

[7] In the relational epistemology we promote, only ratios such as c_0/c_S matter, and not the value of c_0 or c_S taken in isolation.

[8] For example, we write $\sqrt{4} = \pm 2$, so $\left|\sqrt{4}\right| = 2$.

[9] Marking in Φ, rather than in r, is a way of revealing that the space of spatial coordinates r can be understood as a field analogous to another, determined by concrete physical quantities (see our work).

i) We must first assume that we are in the case of the weak-field approximation, that is, that the elementary contributions δM (instead of dM) satisfy

$$\tau = \frac{G\delta M}{rc_0^2} = \frac{G\rho\delta V}{rc_0^2} \ll 1 \tag{12}$$

as seen at a distance r from the observer. We will consider the scale of the universe, where the average density will be of the order of 10^{-26} to 10^{-28} kg/m^3. Let us evaluate the ratio τ by considering the small mass mentioned earlier (in the small unit volume δV) from a distance of the order of one meter; increasing the distance will only decrease the ratio, whose order of magnitude we take to be:

$$\tau = \frac{6{,}67\ 10^{-11}\ 10^{-27}}{1.\ 9.10^{16}} \approx 10^{-54} \quad \ll 1 \tag{13}$$

We observe that we are indeed within the conditions required by relation (12). The weak-field approximation allows us to linearize the metric according to

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{14}$$

where$\eta_{\mu\nu}$ is the metric of a flat (Euclidean) spacetime, and the perturbation is such that

$| \ h_{\mu\nu} \ | \ll 1$. *In this regime, the gravitational contributions add together*.

(ii) We can then linearize Einstein's equations. In the weak limit, the local gravitational potential Φ satisfies

$$\nabla^2\Phi = 4\pi G\rho \tag{15}$$

Where ρ is the local matter density. We perform a summation to calculate the potential as seen by an observer located at a distance r from the origin, using Newtonian superposition. The summation is performed over all mass elements located at a distance $\vec{r}'$ from the origin and at a distance$\vec{r} - \vec{r}'$ from the observer, i.e.,

$$\Phi = -G\int \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|} dV' \tag{16}$$

iii) *Cosmological principle.* We assume that the universe is homogeneous and isotropic on large scales, and thus $\rho = \rho_u =$ constant where ρ_u is the average density of the universe. We will substitute this value into the previous integral in place of ρ(r').

iv) *Copernican principle.* Our perspective on the universe is not privileged[10] and every observer has the same one. We assume that each spherical shell around the observer contributes equally. This amounts to replacing $|\vec{r}-\vec{r}'|$ with r' in (16), and the observer is located at the "center" of the universe.

v) Integration over the scale of the universe. We replace the volume dV in (16) with the spherical shell of volume $4\pi r^2 dr$ [11]. The integral over a horizon of size R_u for the universe is written as

$$\int_0^{R_u} \frac{\rho_u 4\pi r^2}{r} dr \tag{17}$$

and becomes $4\pi\rho_u \int r\, dr$, integrating to $2\pi\rho_u R_u^2$. We thus obtain an average gravitational potential

$$\Phi = -\, 2\pi G \rho_u R_u^2 \tag{18}$$

which yields the following metric coefficients:

$$g'_{tt}(\beta) = a' = 1 + \frac{2\Phi}{c_0^2} = 1 - \frac{4\pi G\rho_u R_u^2}{c_0^2} = \quad (1-\beta) \tag{19}$$

$$g'_{rr}(\beta) = \; b' = \left(1 + \frac{2\Phi}{c_0^2}\right)^{-1} = \left(1 - \frac{4\pi G\rho_u R_u^2}{c_0^2}\right)^{-1} = (1-\beta)^{-1} \tag{20}$$

where

$$\beta = \frac{4\pi G\rho_u R_u^2}{c_0^2} = -\frac{2\Phi}{c_0^2}$$

This parameter is a function of the size R_u of the expanding universe, which itself depends on time, i.e., $R_u(t)$ (at constant density). More generally, β is a function of the product $\rho_u R_u^2$ which can vary depending on the expansion. In summary, β is an expression of the gravitational potential Φ experienced at a point in the universe (similarly, we had noted that the coefficients of the metric S depend on the local gravitational potential). This quantity replaces the distance

[10] Due to the Big Bang, it is as if every point were "at the center" of the universe.
[11] The dummy integration variables r or r', V or V' are equivalent.

r on which the coefficients of the metric S depend [12]. By performing a finite expansion of the values of the metric coefficients and the index in the vicinity of a value of β, we can simulate the effects of deviations from the parameter values obtained under the assumptions of homogeneity and point-of-view independence. For a critical universe, the metric will be strictly constant (within the framework of the simplifications that allowed it to be calculated).

The metric G, obtained from the classical metric S, is understood in an extended sense relative to a standard metric, whose coefficients depend on the coordinates (x, y, z, t) of the point under consideration. For G, there is no longer any need to concern oneself with the coordinates, since we have considered the assumptions of homogeneity and viewpoint invariance: all points are equivalent[13] .

In summary, the G-metric corresponds to a weak-field metric generated by the total mass of the universe. We have explicitly assumed the summation of a gravitational field within the framework of a linearization of Einstein's equations, that is, the principle of superposition of potentials. We have adopted the cosmological assumption of homogeneity and isotropy (homogeneous cosmological density, isotropy around the observer) and have performed the integration up to a cosmological radius R_u. None of these assumptions was explicitly stated, and none was involved in the derivation of the initial metric S.

From this, we obtain a cosmological index n_c such that

$$n_c^2 = \frac{b\prime}{a\prime} \tag{21}$$

$$n_c = \left|\sqrt{\frac{b\prime}{a\prime}}\right| \tag{22}$$

which is written as

$$n_c^2 = \left(1 - \frac{4\pi G \rho_u R_u^2}{c_0^2}\right)^{-2} = \left(1 + \frac{2\Phi}{c_0^2}\right)^{-2} \tag{23}$$

Let

[12] We can use the metric for a "constant" universe as a snapshot, valid for "a certain time," of an expanding universe.
[13] Is this the reason why, unlike in the case of a zero potential at the strict center of a spherically symmetric matter distribution, the value of Φ within the universe is not zero (this question would undoubtedly merit special consideration)?

$$n_c = \left|1 - \frac{4\pi G \rho_u R_u^2}{c_0^2}\right|^{-1} = \left|1 + \frac{2\Phi}{c_0^2}\right|^{-1} \quad (24)$$

Use of the index. In our work, we have shown that a value close to 2.4 is entirely plausible for universes that are reasonable in terms of ρ_u and R_u (densities ranging from 10^{-26} and 10^{-28} kg/m³, or universe radii measured in tens of billions of light-years). The index formula presented here has not changed from our earlier work[14] ; it is now based on a metric G (new compared to S, which was the only one previously cited) under clearly stated assumptions.

3. A cosmological index exactly equal to 2 for a universe at critical density

At this stage, armed with relation (24) for the factor n_c, we could explore —somewhat blindly— the pairs (ρ_u, R_u) that give it the value 2.4. That is not the approach we will take here. In fact, our universe is not just any universe; it has the particular property of being close to the critical density, that is, flat (with zero curvature, neither positive nor negative)[15]. This density (in matter-energy) satisfies, within the standard model

$$\rho_c = \frac{3H_0^2}{8\pi G} \quad (25)$$

where H_0 is the Hubble constant. By definition, we have

$$\rho_u = \Omega\, \rho_c \quad (26)$$

where Ω is the cosmological density parameter. In the critical universe, we will have Ω = 1. Let us introduce ρ_c and Ω into the expression for the index:

$$n_c = \left|1 - \frac{4\pi G (\Omega \rho_c) R_u^2}{c_0^2}\right|^{-1} \quad (27)$$

Then let's substitute the expression for ρ_c . We get:

[14] When taking the square root, we did not account for the absolute value: an index is positive, and the term raised to the second power can be negative. If $a^2 = b^2$, $a = \pm b$, and, if a is assumed to be positive, $a = |b|$. A negative index would have no physical meaning here (as with metamaterials): we are interested in "standard" light propagation. The relationships expressed graphically in our work between $\log\rho_u$ and $\log R_u$ remain unchanged: they must be scaled according to $|n_c|$ and not simply n_c. As for n_c^2, it is given by a square in (23) and is always positive.

[15] The discussion of a critical universe is new compared to our earlier work. In addition to applying directly to our universe, it has the advantage of revealing remarkable simplifications, as the following text will show.

$$n = \left|1 - \frac{3\Omega}{2}\frac{H_0^2 R_u^2}{c_0^2}\right|^{-1} \quad (28)$$

A natural choice for the cosmological radius is the Hubble radius: it corresponds to the most distant points influencing the gravitational field at the observer's location[16], given the expansion of the universe since a time of the order of $1/H_0$, i.e.,

$$R_u \approx \frac{c_0}{H_0} \quad (29)$$

We then have

$$\frac{H_0^2 R_u^2}{c_0^2} = 1 \quad (30)$$

Substituting into the value of the index, we are left with

$$n_c \approx \left|1 - \frac{3}{2}\Omega\right|^{-1} \quad (31)$$

For the gravitational potential Φ, we have (using (31) and identifying the terms in equation (24), for example)

$$\frac{2\Phi}{c_0^2} = -\frac{3}{2}\Omega$$

$$\Phi = -\frac{3}{4}\Omega c_0^2 \quad (32)$$

The index n_c and the potential Φ depend only on the cosmological density parameter Ω (equations 31 and 32). In the previous expressions, they no longer explicitly depend on H_0, the size of the universe R_u, or its density ρ_u. The cosmic index directly measures the overall gravitational density of the universe in the parameter Ω. These results precede those concerning the critical universe and constitute an important point.

For our universe, with $\Omega \approx 1$, we obtain

$$n_c \approx 2 \quad (33)$$

[16] It is also the distance of the horizon receding at speed c_0 according to Hubble's law.

And, for the gravitational potential Φ in the critical universe (Ω = 1)

$$\Phi = -\frac{3}{4} c_0^2 \tag{33}$$

And so, in terms of order of magnitude[17]

$$|\Phi| \approx c_0^2 \tag{34}$$

The value of $n_c \approx 2$ is obtained for the critical universe in a general formula, without any arbitrary adjustments. While the coefficients of the metric and the cosmological index, in their initial interpretation, depend on the parameters of the expanding universe (see (19), (20)), *these parameters are now constant for a critical universe* (regardless of its evolutionary characteristics of density and dimension)[18] .

4. From the critical universe to Mach's principle

The value found at the moment for the cosmological index, equal to 2, approaches the sought-after value of 2.4: this opens the way for us! However, the discussion allowing us to shift n_c from 2 to 2.4 will follow later. In this section, we focus on certain properties of the critical universe: they lead us toward Mach's principle in a new way. This principle "is a conviction formulated by Ernst Mach according to which the inertial (or Galilean) character of a reference frame is determined by the distribution of all the masses in the universe " (Taillet et al., 2013; see also Mach, 1897). There are many ways, sometimes contradictory, to understand this principle, and Einstein himself, who long cited it as the inspiration for his theory of general relativity, formulated it in various ways over time (Einstein, 1922; Hoefer, 1993; Assis, 1999; Fay, 2024; Minazzoli, 2025). Moving from matter to geometry means expressing the coefficients $g_{\mu\nu}$ of the metric when the material content of the universe is given[19] .

A detailed discussion of Mach's principle is beyond the scope of this article. We will choose two interrelated approaches.

[17] We wondered if a factor of 4/3 had not escaped our notice somewhere?
[18] We have a' = -1/2, b' = -2; n^2 = b'/a' = 4, n = 2.
[19] Einstein: "The metric structure must be attributed to a material distribution." According to the cited authors, Einstein also invoked Mach's principle to justify the introduction of the constant Λ; the question was raised regarding the incompatibility of the principle with the choice of Minkowski-type boundary conditions at infinity for the universe.

4.1. Law of Inertia (Galileo, Newton)

The metric G incorporates, in its coefficients, the gravitational potential Φ_{univers} , which arises from all the masses in the universe. This has allowed us to determine the effective speed of light on the scale of the universe, via the index n_c (the condition of zero geodesics). According to the principles of general relativity, we can also use the metric to obtain the law of temporal evolution of free particles (not subject to forces), following their own geodesics. This involves the Christoffel symbols $\Gamma^{\alpha}_{\mu\nu}$, which are functions of the derivatives of the metric coefficients $g_{\mu\nu}$ (where μ, ν, and α refer to the four variables x, y, z, and t):

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}\, g^{\alpha\sigma}(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}) \tag{35}$$

Where we adopt the convention of summing over repeated indices at different heights, and where

$$\partial_\mu g_{\nu\sigma} = \frac{\partial g_{\nu\sigma}}{\partial x_\mu}$$

With these symbols, the geodesic equations are written as:

$$\frac{d^2x^\mu}{dt^2} + \Gamma^{\mu}_{\alpha\beta}\frac{dx^\alpha}{dt}\frac{x^\beta}{dt} = 0 \tag{36}$$

For the critical universe, the coefficients $g_{\mu\nu}$ of the metric G are constant, the $\Gamma^{\alpha}_{\mu\nu}$ are zero, and we have

$$\frac{d^2x^\mu}{dt^2} = 0 \tag{37}$$

This relationship expresses that, in the absence of force, the particle has zero acceleration. It describes a straight line with constant velocity[20] : this is the expression of the law of inertia, Newton's first law; or, alternatively, Galileo's principle of inertia. A local property (law of inertia) connects to a global property (constancy of the gravitational potential of the universe).

The consistency between the property of constancy of the metric coefficients and that of the curvature of the universe can also be noted. The tensor used to calculate the curvature, namely

[20] We will not dwell on two points that would merit a mathematical digression regarding the reference frames used. - The consideration of a time to be specified: the original form of equation (36) involves proper time τ, which we extend here to a "collective" time t. - The lack of specification regarding the type of coordinates (Cartesian or spherical), and the absence of mention of the type of symmetry (spherical) required beforehand.

the Ricci tensor $R_{\mu\nu}$, derived by contraction from the Riemann curvature tensor R^{i}_{klm}, is obtained from the Christoffel symbols and their derivatives. Due to the constancy of the metric coefficients, all terms in the tensor $R_{\mu\nu}$ are zero. The curvature is zero; this is another name for criticality, which ensures that $\Omega = 1$.

4.2. Inertial Mass and Gravitational Mass

Behind the law of inertia lies inertial mass. For a critical universe, the total gravitational potential of the observable universe $|\Phi|$ at a point satisfies $|\Phi| \approx c_0^2$ [21]. The energy embodied by a mass m, and due to the attraction by all the masses of the universe, is $m.|\Phi|$, that is, mc_0^2. This is the inertial mass m_i, and taking into account the value of Φ:

$$m_i \approx m.|\Phi| \approx mc_0^2 \tag{38}$$

This is a way of expressing Mach's principle: local inertia depends on the total mass of the universe in Φ. Inertial mass is a form of potential energy (m is measured in kilograms, $m\,c_0^2$ in joules). One could also refer to it as relational mass

Furthermore, according to special relativity, the energy associated with mass m at rest is $m\,c_0^2$. This is the gravitational mass m_p (i.e., a quantity of matter specific to the object, which could be described as "substantial," as opposed to the relational mass mentioned earlier):

$$m_p \approx mc_0^2 \tag{39}$$

We observe (relations (38) and (39)) that the two masses m_i and m_p are equal, that is

$$\textit{inert mass} \approx \textit{gravitational mass} \tag{40}$$

We find this equality, verifying the consistency of general relativity, which takes it as one of its foundations.

[21] This relationship appears in several discussions related to Mach's principle. In 1953, Sciama proposed: $|\Phi| = \sum \frac{GM_i}{r_i} \sim c_0{}^2$ as a quantitative formulation of Mach's principle, where the summation is taken over all masses M_i located at a distance r_i from the observer (see also Sciama, 1969; Brans & Dicke, 1961).

4.3. Discussion: Mach's principle

Thus, our approach, based on the G-metric, links two laws that are often considered in separate contexts (even though we assume a deep connection between them): the law of inertia and the law of equivalence between inertial mass and gravitational mass. In his equation, Einstein linked the parameters of geometry to matter, but he did so locally, without referring to the total mass of the universe (Einstein expressed regret about this, saying that he had not fully implemented Mach's principle[22]). The relativistic scale ${c_0}^2$ (which allows us to determine the energy content of mass) and the global gravitational potential of the universe are of the same order of magnitude: relativity thus appears to be deeply linked to the gravity of the entire universe.

We have established a quantitative, operational link between general relativity and Mach's principle (taken in a broad sense). The latter is no longer merely a philosophical principle: the coefficients $g_{\mu\nu}$ of the metric are effectively determined by the material content of the universe. The link between Mach and Einstein is made possible by the inclusion of the gravitational potential Φ in the metric G. This corresponds to a linearized approximation of general relativity, applied to a homogeneous cosmological distribution (weak field; superposition of potentials; cosmological homogeneity and isotropy). All of this holds for a critical universe.

5. Reciprocal: From Mach's principle to the critical universe. The universe is flat by nature

5.1. Reciprocal

The property of flatness (or criticality) of the universe supports the review we have just conducted. The previous section can be summarized in the following inference:

$$\textit{Critical universe} \rightarrow \textit{Mach's principle} \tag{41}$$

That is to say: the criticality of the universe has allowed us to make the connection between the local and the global, fulfilling a wish of E. Mach. But one might ask: how is it that our universe is close to critical density? Is it a coincidence, due to very specific initial conditions of mass

[22] Assis (1999) discusses the points of disagreement between Einstein and Mach on this issue.

and energy at the time of the Big Bang? Let us pause on this point, which has long been raised by physicists. We answer it with a reciprocal to the previous inference in the form

$$\textit{Mach's Principle} \rightarrow \textit{Critical Universe} \tag{42}$$

That is to say: the link between the local and the global is a way of expressing criticality. We can provide two mathematical justifications, using the properties just established for a Machian world.

According to the first, let us assume the equality of inertial mass and gravitational mass (see equations (38) through (40)), written in the form

$$m.|\ \Phi\ | \approx mc_0^2 \tag{43}$$

From this we obtain $|\Phi| \approx c_0^2$. Knowing that $|\Phi| = \frac{3}{4}\ \Omega c_0^2$ (relation 31), which we equate to $|\Phi| \approx \Omega c_0^2$, we deduce that $\Omega \approx 1$ [23] : the universe is critical.

According to the second mathematical justification, we perform a calculation of the density ρ. We have the relation (*see* 18 and 34)

$$2\pi\rho G R^2 = c_0^2 \tag{44}$$

which is one way of writing the potential Φ. We add the relation $c_0 = H_0 R$, which states that the size R of the universe is identified with its horizon, which recedes at a speed of c_0 (H_0 is the Hubble constant). From this we deduce that

$$\rho = H_0^2 / 2\pi G \tag{45}$$

which we can compare to the expression for the critical density within the standard model, namely

$$\rho_c = \frac{3H_0^2}{8\pi G} \tag{25}$$

We observe that these last two expressions are of the same order of magnitude. We conclude that *we are dealing with a critical universe*.

[23] The calculations must be repeated with $\Omega = \rho_b/\rho_c$, restricting ourselves to baryonic matter (subscript b), see Section 7. In the calculation, we have assumed that the parameter m is available, identical in both forms of mass, inertial and gravitational (we do not discuss the aspects of circularity hidden therein).

5.2. Platitude Conjecture

In light of these mathematical results, we are led to seek a deeper physical meaning for these properties. To this end, we state the following flatness conjecture:

Our universe appears to us naturally as flat (46)

Or, in other words: it is *essentially,* or *identically, flat*. By this, we mean that *criticality "simply" reflects a way of speaking about the world,* about the interconnections between its phenomena and the quantities that appear within them.

Let us elaborate on this point of view.

The conjecture fits into what we believe to be the "normal" epistemological functioning of physics. It is relational, based on the use of ratios: only these ratios have meaning, not the individual terms that compose them. Thus, a speed, such as that of light, is measured in comparison with another speed v, of gravitational origin, in a ratio of the form v/c, whose denominator has been stabilized. In our terrestrial context, the ratio is 3.10^8. Very generally speaking, since the speed v itself depends on a density distribution at different distances, it is the value of a ratio of the form $G\rho R^2/c^2$that matters, where the various terms can be varied, provided that it remains constant (Guy, 2025b) [24] . A speed c that would relate to the meter and the second as independent of phenomena makes no sense. Conversely, a "gravitational" speed cannot be evaluated independently of that of light. For the universe taken as a whole, on the surface of which a test mass would be placed, the speed of the receding horizon is c_0, whose square is equal to the gravitational potential. Based on the definition of speeds, it is, more fundamentally, the concepts and measurements of space and time that are constructed as related to light and masses. These propositions are consistent with general relativity and Mach's principle regarding the relationship between matter and motion. They include the quantities we have previously discussed, which refer to the characteristics of our universe and which, *ultimately*, reveal the link between Φ and c_0^2 at criticality.

In this framework, we cannot answer questions such as: - which do we consider first: masses, or velocities, or their relationships? - Is it the expansion velocity that varies, or the mass of the

[24] For example: variation of the constant G with distances to influential masses (Paturel, 2023); covariation of densities ρ and dimensions R, or scale invariance (Gueorguiev & Maeder, 2022; Maeder, 2023; Maeder and Courbin, 2024; Vaucouleurs, 1971); as well as other covariations, such as that of G and c, etc., examined in our work (Guy, 2025b). We can agree that Mach's principle conceals many perspectives!

universe? - Does the speed of a given object actually vary, or is it merely our perception of it—due to the potentially variable speed of light—that leads us to believe so? [25] . Or again: - Can we distinguish between the inhomogeneity in the distribution of object sizes and densities, and the spatial variation of the constants G and c? The unifying nature of gravity (the same effect results from a nearby, low-mass object or a distant, higher-mass one), or the foundations of quantum physics, fall within the same perspective of a unity of reality, prior to its division into different viewpoints by human observers. This division is made with an irreducible element of convention, gradually linking the various manifestations of the connections between matter and motion. Yet the illusory conviction always returns that one could judge a mass (the universe) and a velocity (expansion) independently in order to determine criticality!

All in all, our criticality conjecture does not reveal a mysterious coincidence linking two quantities that are unrelated to one another: on the one hand, the speed of light, and on the other, the gravitational potential generated by the mass of the universe. The balance of the two terms in this relation expresses a physical law implicit in the correspondence between forces of different origins, one electromagnetic in nature, the other gravitational[26] .

Discussion of the standard understanding of flatness: what role does inflation play?

Our explanation of the universe's flatness seems physically intuitive to us and faithful to Mach's spirit. But within the standard framework, flatness remains a mystery to the authors. The ΛCDM model interprets flatness (or quasi-flatness) as the consequence of an early phase of cosmic inflation. This dynamically drives the curvature parameter toward zero, thereby explaining the observed spatial flatness of the universe. The critical density then has no deeper dynamic significance beyond its role as a geometric threshold separating open, flat, and closed universes. The approach developed in our work therefore suggests a different perspective. The condition naturally follows from the requirement that the global gravitational potential Φ is of the order of c_0^2. The observed quasi-critical density of the universe does not require a specific initial condition (such as inflation), but rather reflects a condition of global coherence of the physical world.

[25] Similarly, we cannot know whether it is the film that is being projected faster or slower, or whether it is the actor who is mimicking the greater or lesser speed. We might say: it is undecidable; we can consider several "options."
[26] This is a reason to say that the question of the stability of the critical universe, and of the introduction of the constant Λ, does not arise.

6. Possible increase of n_c from n = 2

6.1.Leaving the critical region?

Let's return to our cosmic refractive index. We said, *"We need 2.4, not just 2!"* There are various ways to achieve this increase. We could decrease Ω slightly: for example, for a value of $\Omega = 0.95$, we see that $n_c = 2.35$ (see equation (31)). Various pairs (ρ_u , R_u) would work; this is not a problem, as we have already explored it (Guy, op. cit.). Starting from the critical universe, we are in a region that is very sensitive to an increase in n_c: this parameter belongs to the interval $[2, +\infty]$, since the denominator in the formula for n_c is close to zero at criticality—the value 2.4 is very close!

It turns out that all of these different paths would lead us out of the critical region. Should we do so? In the context described in the previous section, the criticality appears to be robust. We believe it is indeed so, as the authors also point out; the value $\Omega = 1$ is well established (particularly through the study of the cosmic microwave background), even though it is calculated within the framework of the Friedman equation, where a constant Λ and dark matter have been incorporated. *Let us remain close to the critical region.* Returning to the discussion that led us to the calculation of the cosmological index and its value of 2 under critical conditions, we can then conclude: if we wish to increase the value of the index from 2, it is rather the basic assumptions of the cosmological model that must be challenged. The homogeneity of the universe on large scales is a first approximation (as assumed by the standard FL metric)[27]. We know, however, that the universe is highly heterogeneous: empty regions (accounting for between 50 and 80% by volume) with a density ρ_v lower than the average ρ_u ; and dense, structured regions (walls, filaments, galaxy clusters, galaxies…) with higher ρ_s (occupying the remaining 20%, or more).

Let us thus consider the first-order case under the homogeneous and critical assumptions, and, in the second-order case, add the effect of inhomogeneities. We expect dense regions to slow down light more and increase the index. However, it can be easily shown that, if the dependence $n_c(\rho)$ is linear, the effect of inhomogeneities is zero, as long as the total mass is conserved. On the other hand, if the dependence is nonlinear, which is the case for our distribution $n_c(\rho)$,

[27] According to Fay (2024), Mach's principle and the law of inertia require the universe to be homogeneous, which suggests that their validity is generally approximate.

overdensities increase the light delay—that is, the refractive index—more than underdensities compensate for it (variance correction).

6.2. A calculation

Let's perform a calculation to show that this hypothesis *("n_c can easily increase from 2 to 2.4 due to inhomogeneities")* is valid. Our assessment is approximate, barely realistic, simply seeking orders of magnitude for density variations—even mild ones—that lead to an increase in the index. To do this, we perform a limited expansion of the index as a function of density ρ, around the characteristics of the critical universe (in particular its density ρ_u and its dimensions). We assume that the contributions from dense regions and empty regions, characterized by their sizes, can be added separately (using this linearization method, we can only consider modest variations). We also assume that the volume fractions of the voids f_v and the structures f_s are given independently of the choices regarding the densities; strictly speaking, this is incorrect, since the way in which ρ is grouped into intervals influences the distribution f_v/f_s[28] .

Conservation of mass. Let V_v and V_s be the volume fractions of the voids and the structures, respectively, and let ρ_v and ρ_s be the corresponding densities. We are considering a given volume of the universe, and we will set $V_v = f_v = 0.8$ and $V_s = f_s = 0.2$. In this scenario, the voids occupy four times as much volume as the dense regions. The conservation of mass is expressed as

$$\rho_s \; V_s \; + \rho_v \; V_v \; = \text{const} \tag{47}$$

or, by differentiating and using the chosen proportions

$$0.2 \; \delta\rho_s \; + 0.8 \; \delta\rho_v \; = 0 \tag{48}$$

Where $\delta\rho_s$ and $\delta\rho_v$ are to be calculated relative to the reference state. We are in the vicinity of the critical density, which is of the order of 9.10^{-27}kg/m³ . We consider this to be one of the reference quantities for the critical universe. We will seek new densities ρ' that differ from the initial uniform density ρ_u, which we denote as $\rho = 9.10^{-27}\,\rho'$. Hereafter, the notation ρ will be retained for ρ'.

[28] Seeking the ratio f_v/f_s as an unknown is possible, but leads to calculations that are too complicated for our initial approach.

Variation of the total index. Let us start with the nonlinear law (see (24)), which we write in the form

$$n_c(\rho) = \frac{1}{|1-\alpha_{Ru}\rho'|} \tag{49}$$

With the coefficient α_{Ru} for the reference state of critical universe (u subscript refers to universe, R to reference state, but Ru may also be referring to universe radius still at criticality). Let us differentiate n with respect to ρ by summing two terms (considered as partial derivatives), distinguishing them both by the densities ρ' and by the corresponding radii, modifying the factor α_{Ru} depending on whether we are in a zone v of volume V_v (or of corresponding radius R_v) or in a zone s, of volume V_s (R_s). We write:

$$\delta n = \frac{\partial n}{\partial \rho_v}\delta\rho_v + \frac{\partial n}{\partial \rho_s}\delta\rho_s \tag{50}$$

Considering the global dimension of the universe R_u in the factor α_{Ru}, R_s and R_v are perturbations of it. We have expressed the volume sizes in terms of the ratios $f_s = V_s / V_u$ and $f_v = V_v / V_u$ (equivalent to a universe of unit volume) to be related to the radii (in the expression of the index).

For the universe as a whole, and under critical conditions with the relevant cosmological branch[29] (negative before the absolute value), the denominator of n_c is equal to -1/2 and the reference factor α_{Ru} is equal to 3/2 (with Ω = 1). And n_c is written as

$$n_c = \frac{1}{|-1/2|} = 2$$

δn is expanded in the vicinity of the reference value α_{Ru}

$$\alpha_{Ru} = \frac{4\pi G\rho_u R_u^2}{c_0^2} = \frac{3}{2}$$

The derivatives $\frac{\partial n}{\partial \rho_v}$ and $\frac{\partial n}{\partial \rho_s}$ are obtained by differentiating n with respect to ρ (at first-order for an initial approximation):

$$n = \frac{1}{(1-\beta)} \tag{51}$$

[29] We speak of branches in the calculation of the index for the two choices of sign (+ or -) of the positive or negative quantity whose absolute value is taken.

$$\delta n = \frac{\alpha_{Ru}}{(1-\alpha_{Ru})^2}\delta\rho = \alpha_{Ru}n^2\delta\rho \qquad (52)$$

And we linearize by summing the two contributions

$$\delta n = \frac{\partial n}{\partial \rho_v}\delta\rho_v + \frac{\partial n}{\partial \rho_s}\delta\rho_s = \alpha_{Rv}n^2\delta\rho_v + \alpha_{Rs}n^2\delta\rho_s \qquad (53)$$

Where the unknowns are $\delta\rho_v$ and $\delta\rho_s$ respectively.

We want to see if it is possible to go from n = 2 to n = 2.4, i.e., an increase in $\delta n/n$ equal to 0.4/2 = 20% around n = 2. We obtain

$$\frac{\delta n}{n} = \alpha_R n\delta\rho = 2\alpha_{Rv}\delta\rho_v + 2\alpha_{Rs}\delta\rho_s = 0{,}2 \qquad (54)$$

If we express the volumes in spherical form, with radii R_s and $R_{(v)}$,we will have for the latter as functions of V_s and V_v (or f_s and f_v):

$$R_v = (\frac{3}{4\pi}0{,}8)^{1/3} \qquad (55)$$

$$R_s = (\frac{3}{4\pi}0{,}2)^{1/3} \qquad (56)$$

And thus, with $\alpha_{Ru} = 3/2$,

$$\alpha_{Rs} = \alpha_{Ru}\ (R_s)^2 = 3/2(R_s)^2 \qquad (57)$$

and

$$\alpha_{Rv} = \alpha_{Ru}(R_v)^2 = 3/2(R_v)^2 \qquad (58)$$

Let us substitute the values of R_v and R_s given in (55) and (56)

$$\frac{\delta n}{n} = \alpha_{Rv}n\delta\rho_v + \alpha_{Rs}n\delta\rho_s = 2\times\ 0{,}335\delta\rho_v + 2\ \times 0{,}134\ \delta\rho_s\ = 0{,}2 \qquad (59)$$

We have thus written two first-degree equations in $\delta\rho_v$ and $\delta\rho_s$, namely (48) and (59). The solutions are

$$\delta\rho_v = -0.49 \quad \text{and} \quad \delta\rho_s = 1.96 \qquad (60)$$

This implies a decrease of 0.49% for the empty regions and an increase of 1.96% for the structured regions; or, alternatively $\rho_v = 9 \times 10^{-27} \times 0.49 = 4.41 \times 10^{-27}$ kg/m³ and $\rho_s = 9.\ 10^{-27} \times 1.96 = 1.76 \times 10^{-26}$ kg/m³ , resulting in an overall increase in the index from 2 to 2.4. These are somewhat lower contrasts, but close, compared to what is known, where one might expect a multiplicative or divisive factor on the order of 10. This approach, which attributes the effects of dark matter and dark energy (via n_c in our case) to the inhomogeneities in the distribution of mass in the universe, is consistent with the work of Buchert (2012, 2018).

The simplified calculations above suggest that the growth of inhomogeneities during expansion could be responsible for an increase in the cosmic index, in the portion associated with this process. This would help to mimic the evolution of the quantity and distribution of dark matter and dark energy over time. One can also imagine that, for objects of the same age, and depending on the context in which an object is located (in a more or less dense, more or less heterogeneous region), the postulated amounts of dark matter and dark energy may vary. These questions require the examination of extensive data and are beyond the scope of this article.

7. Adjustment of calculated and measured baryonic matter densities

If we no longer need dark matter or dark energy, we are left with baryonic matter alone, and, for $\Omega = 1$, its density must be critical. Is this compatible with the various mathematical and observational constraints? The amount of baryonic matter is estimated in two different ways, which converge in the Standard Model: - the one based on the distribution of the various Ω_i, adjusted for the proper functioning of the Friedman equations; let's call this $\rho_{b\Lambda CDM}$; - the one evaluated by “direct” counting or measurement in the galactic and intergalactic medium: let us simply call it ρ_b . In the context in which we find ourselves, and given the question of multiple contributions to the total density, we must rely first on the direct estimate of ρ_b . It is this value that now provides us with the critical density. If $\rho_{c\Lambda CDM}$ was the critical density estimated within the standard model, we seek to ensure that ρ_b can become a critical density of baryonic matter, within the framework of a new model based on the G metric, namely ρ_{bcG}. Within the framework of a revised model, the Hubble constant H_0 must be divided by a factor of 2.4, since it is the set of velocities that construct the Hubble diagram that must be scaled. The parameter $\rho_{c\Lambda CDM}$ must then be divided by the factor $(2.4)^2 = 6$, as seen by satisfying the critical relation

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

In the new understanding, the critical value of ρ_{bcG} corresponds to the critical value of the ΛCDM model divided by 6, that is

$$\rho_{bcG} = \rho_{c\Lambda CDM} / 6 \qquad (61)$$

The critical density $\rho_{c\Lambda CDM}$ is currently estimated to be 9×10^{-27}kg/m³ . Dividing this by 6 yields a value of the order of 10^{-27}kg/m³. Given that the density of baryonic matter ρ_b is estimated to be on the order of 5×10^{-28}kg/m³, with an uncertainty of a few percent, we see that the orders of magnitude are consistent. We will say no more on this for now, pending a more complete rewriting of the models.

8. Comparison between the FRLW metric and the G metric

Here we offer some preliminary considerations on expansion models and the comparison between the G metric and the FL metric.

8.1. Comparison of the two metrics

How do the two metrics accommodate the expansion of the universe, given that G is not designed for this from the outset, unlike FL? In standard cosmology, the dynamics arise from the scale factor $a(t)$. At first glance, the G metric provides a static representation of the universe: authors often take this approach, referring to it as average density or radius (defining a global gravitational potential). With G, which is based on the equations of general relativity, expansion is not denied: it can be read in the properties of the embedded index n_c. One can indeed simulate expansion, via the size of the universe R, with equal total mass. The index can be understood as a function n(R), reflecting the expansion, without creating it (see Guy, 2025a). This is what will allow us to compare n and a(t). Speaking of the index, the comparison is local and is made for light in the expression of the metric by $ds^2 = 0$. For the G metric, we write

$$a'(R)c_0^2\, dt^2 - b'(R)dr^2 = 0 \qquad (62)$$

which, given the definition of the index n_c, can be written as

$$c_0^2\, dt^2 - n_c^2\, dr^2 = 0 \qquad (63)$$

For the FL (or FLRW) metric, we have a similar expression

$$c_0^2\, dt^2 - a^2\,(t)dr^2 = 0 \qquad (64)$$

Where a is the scale factor. By comparing the two expressions, we see that the index n_c plays the momentary role of the scale factor a(t) [30] . But the correspondence ends there. For the

[30] Note: we do not use here what is called in the literature the optical metric (Gordon, 1923, and others). These authors incorporate Maxwell's equations into general relativity.

evolutions of R(t) and a(t) have different signs during the expansion (Guy, 2025a), and, for a value of ds^2 different from zero, we can no longer make the previous identification. We then have for G

$$ds^2 = a'(R)c_0^2\, dt^2 - b'(R)dr^2 \quad (\neq 0) \tag{65}$$

to be compared for the FL metric with

$$ds^2 = c_0^2dt^2 - a^2(t)dr^2 \qquad (\neq 0) \tag{66}$$

To introduce an index in G, we must divide ds^2 by the factor a'. We then simultaneously obtain a new ds' and a new dt' according to

$$ds'^2 = \frac{ds^2}{a(R)} = c_0{}^2dt^2 - \frac{b(R)}{a(R)}dr^2 \tag{67}$$

The two metrics do not correspond to each other[31] .

FLRW Metric and the Speed of Light

For the G metric, criticality will result in a series of direct consequences that we have explored. In contrast, for the FLRW metric, the effects of criticality are more hidden. The two metrics also do not treat the speed of light in the same way. For the FLRW metric, the parameter c does not shift from its value c_0; the question of its possible variation does not arise. We do not speak of a refractive index; c_0 becomes a structural factor whose primary physical meaning has disappeared. The scale factor a(t) and comoving distances also make it difficult to approach the effective speed of light, as one can do with the G-metric. With the use of the factor a(t), we allow for a dissociation between an expanding space—conceivable on its own (without its material content)—and comoving distances measured between celestial bodies. Within the framework of understanding we have presented, considering the propagation of light in a space devoid of matter makes no sense: it is a useful fiction. Finally, unlike the G-metric, the FLRW-metric does not allow for an easy consideration of the covariations of physical quantities (ρ, R, c, G) and what we have grouped under the name of scale invariance (see Section 5.2 and Note 23).

[31] Another way to proceed further would be to change the values of proper time or to define a "comobile" index?

Conclusion

Let's take a look back at how we got here. As we have seen, the value $c_0$of the speed of light—which is used in cosmological calculations (Doppler effect, gravitational lensing, etc.)—could be reduced by a factor of 2.4. This would, on average, resolve the problems posed by astrophysics (see Guy, 2025a, and Appendix 1).

The prediction of this factor is done in two steps. First, the calculation of a cosmological refractive index n_c, via a new metric G, incorporating parameters of the universe's density and size. It is constructed based on the assumptions we have outlined, in particular that of the cosmological principle (homogeneity of the universe). For the critical universe in which we find ourselves, the index is equal to 2. In a second step, we consider the vicinity of this solution, taking into account the effect of inhomogeneities: at second order, these are likely to increase n_c.

The inclusion of the gravitational potential induced by the mass of the universe in the metric coefficients allows for a new perspective on Mach's principle, now framed in terms well-suited to general relativity: the local geometry is linked to the global matter. This is the case in the critical universe. Conversely, this led us to state the conjecture that *the universe is identically flat: one cannot describe (or conceive of) its velocity characteristics and its mass characteristics separately*.

We have drawn attention to the fact that our approach has remained semi-quantitative, addressing the expansion of the universe through a series of frozen snapshots, and without adopting the models at stripped of their dark matter and dark energy content. As it stands, the proposed picture nevertheless gives us hope of comprehensively alleviating a significant set of difficulties presented by the Standard Model, offering numerous avenues for future research.

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Appendix 1 An optical index of 2.4 to shed light on the dark side of cosmology?

We summarize here, in a condensed and simplified form[32], a number of issues currently problematic in cosmology: we believe they could be resolved by the approach proposed in this article. Detailed explanations can be found in the author's works, particularly Guy (2025a). The proposal to reduce the speed of light on a cosmological scale by a factor of 2.4 compared with its standard value in a vacuum may give rise to reservations; but we must not lose sight of the scale of the problem we face (95 per cent of the matter and energy in the universe are unknown). We need a solution that is up to the task!

Dark matter: spiral galaxies. The rotational speeds of galaxies estimated using the Doppler effect are overestimated; if we assume that the speed of light reaching us is lower, the missing mass is no longer necessary.

Dark matter: gravitational lensing. The deflection angle depends on the speed of light: using its standard value overestimates the mass of the influencing matter.

Dark matter: cosmic microwave background (CMB). The temperature of the cosmic microwave background must be lowered (the expansion effect overestimates it). This reduces density fluctuations and the need for dark matter.

Dark energy. There is no dark energy; this is yet another expression of the correction for an overestimated speed of light. We add here that the effect of the universe's inhomogeneities on the cosmological refractive index also eliminates the need to invoke dark energy[33] .

Cosmological constant / quantum vacuum energy. The cosmological constant is a corrective fiction: the enormous discrepancy with quantum vacuum energy does not point to a genuine physical problem.

Variation of the cosmological constant over time. The dark energy effect depends on the cosmological refractive index, which decreases with expansion.

Hubble tension. The Hubble constant H_0 must be divided by a factor of approximately 2.4. The confidence intervals from the various methods of its determination overlap. The calculation using the cosmic microwave background requires less dark matter: the value of H_0 obtained through this method increases and converges with that obtained from the local environment.

[32] The reality is certainly more complex!
[33] Dark energy is also necessitated by the study of the CMB (cosmic microwave background). The characteristics of the CMB itself must be revised within the framework of the present approach.

S8 tension. The inhomogeneities of the universe are reduced when larger volumes are considered (given that the S8 parameter is calibrated to H_0, which must be reduced).

"Impossible" galaxies and black holes. The universe is older than the age given by the standard model. The discrepancy in the behavior of the scale factor a(t) between our modified model and the Standard Model significantly affects the early stages (the nonlinearity between the redshift z and the scale factor differentiates the discrepancies and allows for additional time at the beginning of the Big Bang; see the work of Gupta (2023), who explores this discussion within a different explanatory framework).

The Milky Way's scarcity of dark matter. Velocity measurements based on parallax (rather than the Doppler effect) do not require us to adjust the speed of light, which provides us with this information.

Inflation. Neither time nor space exists "on their own"; they rely on the speed of light, which was much slower at the beginning of the Big Bang. Compared to our standards, the effect of enormous spatial expansion in a short time is understandable. Inflation contributes to the flatness of the universe, but we can understand the latter without it, through its "natural" criticality.

The multiverse. *Argument for*: the universe's horizon isolates it from other possible worlds. *Against*: one cannot invoke this to claim that the laws of physics are one way here and different elsewhere: the laws are not strictly imposed by nature; they are to some extent "decided" by the physicist (see Poincaré's understanding).

Flatness. This work allows us to add this point to the list. Flatness (or criticality) is not a property resulting from a surprising coincidence of the initial conditions of the universe. It expresses the inevitable convergence of physical quantities such as the speed of light, the gravitational constant, and the properties of density and dimensions of the universe.

Mach's principle, inertia, and the equality of inertial mass and gravitational mass. This point is also added. The understanding of the cosmological index within the G-metric (involving the gravitational potential of the universe's masses) gives Mach's principle an operational significance rather than merely a philosophical one.